

Ec 11 Homework 4
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CALTECH



1. An investor discounts future profits at 5% per year. Suppose a stock pays \$1 in dividends after one year, growing 1% each year thereafter. How much should the stock be worth today?
2. You are buying a \$20,000 car. If you make monthly payments of \$1000, how long does it take you to pay off the debt if the interest rate is 1% per month? How does this change when the interest rate drops to $\frac{1}{2}\%$?
3. A bond is a financial instrument that pays a fixed amount, called the face value at a maturity date. Bonds can also pay out fixed payments called coupons in regular intervals up until the maturity date. Suppose a bond with face value \$1,000 sells for \$900 on the market and has annual coupon payments starting a year from today up until its maturity date 10 years from now. What is the coupon rate? Assume $r = 10\%$.
4. The real return on stocks averages about 4% annually. Over 40 years, how much will \$1000 invested today grow?
5. You have invented something. You can sell it now for \$1M, and work at something else, producing \$75,000 per year for ten years. (Treat this income as received at the start of the year.) Alternatively, you can develop your invention, which requires working for ten years, and will net \$5 million ten years hence. For what interest rates are you better off selling now? (Please approximate the solution.)

1. An investor discounts future profits at 5% per year. Suppose a stock pays \$1 in dividends after one year, growing 1% each year thereafter. How much should the stock be worth today?

The present value is:

$$\begin{aligned}
 PV &= \frac{1}{1.05} + \frac{1 \cdot 1.01}{1.05^2} + \frac{1 \cdot 1.01^2}{1.05^3} + \dots = \frac{1}{1.05} \left(1 + \frac{1.01}{1.05} + \frac{1.01^2}{1.05^2} + \dots \right) \\
 &= \frac{1}{1.05} \left(\frac{1}{1 - \frac{1.01}{1.05}} \right) = 25
 \end{aligned}$$

So, the stock should be worth \$25.

2. You are buying a \$20,000 car. If you make monthly payments of \$1000, how long does it take you to pay off the debt if the interest rate is 1% per month? How does this change when the interest rate drops to ½%?

At 1% per month, we have

$$20000 = PV = \frac{1000}{.01} \left(1 - \frac{1}{1.01^n} \right)$$

This solves for $n = 22.43$ months.

At ½% per month, we have

$$20000 = PV = \frac{1000}{.005} \left(1 - \frac{1}{1.005^n} \right)$$

which solves for $n = 21.12$ months.

3. A bond is a financial instrument that pays a fixed amount, called the face value at a maturity date. Bonds can also pay out fixed payments called coupons in regular intervals up until the maturity date. Suppose a bond with face value \$1,000 sells for \$900 on the market and has annual coupon payments starting a year from today up until its maturity date 10 years from now. What is the coupon rate? Assume $r = 10\%$.

We solve:

$$900 = C \sum_{i=1}^{10} \frac{1}{(1.10)^i} + \frac{1000}{(1.10)^{10}}$$

Giving $C = 83.725$

4. The real return on stocks averages about 4% annually. Over 40 years, how much will \$1000 invested today grow?

$$FV = \$1000(1.04)^{40} = \$4,801.02$$

5. You have invented something. You can sell it now for \$1M, and work at something else, producing \$75,000 per year for ten years. Alternatively, you can develop your invention, which requires working for ten years, and will net \$5 million ten years hence. For what interest rates are you better off selling now?

The future value of taking the \$1M is

$$\$1M(1+r)^{10} + \$75K \sum_{i=1}^{10} (1+r)^i$$

This is larger than \$5M if $r > 13.11\%$.

Alternatively, the inequality for the present value of the amounts is

$$\$1M + \$75K \sum_{i=0}^9 \frac{1}{(1+r)^i} > \frac{\$5M}{(1+r)^{10}}$$

Multiply both sides of this inequality by $(1+r)^{10}$ and the expression is the same as the one above, so that again you should sell now if $r > 13.11\%$.