CALTECH

Ec 11 Final Examination
Professor R. Preston McAfee
Winter 2006

Instructions: Open book, open notes, calculators and computers are OK for computations or to read class materials, no collaboration.
Partial credit will be assigned. Please show your work.
You may take this test during any consecutive 4 hour period.
Due Wednesday, March 16, by 5:00 PM. Please deposit in Box outside Baxter 100.

1. (15 points) A firm discounts future profits of a project at 17%. The firm considers investing in a project that will produce profits of $1M in the first year, and grow at either 5% or 10% annually thereafter, and end after the tenth year is complete. If the two growth rates have equal likelihood, what is the project NPV? Is it higher, or lower, than the NPV with a known growth rate of 7½%?
2. (20 points) Consider a monopolist with constant marginal cost facing demand with constant elasticity greater than one. A per unit tax of t is imposed on the monopolist. By how much does the price rise?
3. (20 points) Consider an Edgeworth box, and two goods X and Y. Consumer 1 has utility $\sqrt{x_1}+\sqrt{y_1}$ and consumer 2 has utility $\sqrt{x_2}+\sqrt{y_2}$, where x_i is the consumption of good X by consumer i, and y_i is the consumption of Y by consumer i. Consumer 1 has an endowment of (1,0) for the two goods, while consumer 2 has an endowment of (0,2).
   a. For any given price for good 1 (in terms of good 2), compute the quantity demanded or supplied for good 1 and good 2 by each consumer.
   b. Compute the contract curve, and find the formula for the individually rational segment of the contract curve.
   c. What price or prices support an equilibrium of the system?
4. (20 points) (20 points) A risk averse person has utility of wealth x of $u(x) = x^a$ and a current wealth level of $100,000. What is the most the person would pay for insurance against a 50% chance of a $10,000 loss. (Don't include the expected loss of $5000, but the premium only.)
5. (10 points) Find all the equilibria of the following game:

		Woman	
		Baseball	Ballet
Man	Baseball	(2,3)	(1,0)
	Ballet	(0,1)	(3,2)

The following questions can be adequately answered with short answers, no more than two or three lines.

6. (5 points, short answer) Incoming college students assigned to dorm rooms with roommates that drink alcohol obtain 10% lower GPAs on average, when compared to those with roommates that do not drink alcohol. How would you use economic analysis to improve the assignment of students to dorm rooms?
7. (5 points, short answer) In an antitrust analysis, how would you determine if Sprite is in a market containing colas?
8. (5 points, short answer) In a setting where incentives matter, does an increase in risk reduce the efficiency of production?

Answers

1. (15 points) A firm discounts future profits of a project at 17%. The firm considers investing in a project that will either produce profits of $1M at the end of the first year, and growing either 5% or 10% annually thereafter. If the two growth rates have equal likelihood, what is the project NPV? Is it higher, or lower, than the NPV with a known growth rate of 7½%?

At 5%, the return is

$$PV = \frac{1}{1.17} + \frac{1.05}{1.17^2} + \frac{1.05^2}{1.17^3} + ... + \frac{1.05^9}{1.17^{10}} = \frac{1}{1.17}\left(1 + \frac{1.05}{1.17} + \frac{1.05^2}{1.17^2} + ... \frac{1.05^9}{1.17^9}\right) = 5.509M$$

At 10%, the return is

$$PV = \frac{1}{1.17}\left(1 + \frac{1.1}{1.17} + \frac{1.1^2}{1.17^2} + ... \frac{1.1^9}{1.17^9}\right) = 6.577M$$

The average is $6.043M. At 7.5%, the NPV is less, at $6.013M

2. (20 points) Consider a monopolist with constant marginal cost facing demand with constant elasticity greater than one. A per unit tax of t is imposed on the monopolist. By how much does the price rise? Compute the dead weight loss of taxation.

The monopoly price is $\frac{\varepsilon}{\varepsilon - 1}$ times marginal cost. Since a per unit tax works like a marginal cost, the after tax price is $\frac{\varepsilon}{\varepsilon - 1}(mc + t)$. Thus the price rises by $\frac{\varepsilon}{\varepsilon - 1}$ times the tax.

3. (20 points) Consider an Edgeworth box, and two goods X and Y. Consumer 1 has utility $\sqrt{x_1} + \sqrt{y_1}$ and consumer 2 has utility $\sqrt{x_2} + \sqrt{y_2}$, where x_i is the consumption of good X by consumer i, and y_i is the consumption of Y by consumer i. Consumer 1 has an endowment of (1,0) for the two goods, while consumer 2 has an endowment of (0,2).
   a. For any given price for good 1 (in terms of good 2), compute the demand for good 1 and good 2 by each consumer.
   b. Compute the contract curve, and find the formula for the individually rational segment of the contract curve.
   c. What price or prices support an equilibrium of the system?

a. Given a price p for good 1 in terms of good 2, consumer 1 maximizes $\sqrt{x_1}+\sqrt{y_1}$ subject to $px_1 + y_1 = p$, since the consumer has an endowment of 1 unit of x_1. Thus, $p(1-x_1)= y_1$, and the consumer maximizes $\sqrt{x_1}+\sqrt{p(1-x_1)}$, which gives a first order condition is $0 = ½(x_1)^{-½} - ½\sqrt{p}(1-x_1)^{-½}$, and thus $0 = (1-x_1)^{½} - \sqrt{p}(x_1)^{½}$, or $0 = (1-x_1) - p(x_1)$ and $x_1 = \frac{1}{1+p}$ and $y_1 = p(1-x_1) = \frac{p^2}{1+p}$. Similarly, consumer 2 maximizes $\sqrt{x_2}+\sqrt{y_2}$ subject to $px_2 + y_2 = 2$ since the consumer has an endowment of 2 units of x_2. Thus, $2-px_2= y_2$, and the consumer maximizes $\sqrt{x_2}+\sqrt{2-px_2}$, which gives a first order condition is $0 = ½(x_2)^{-½} - ½p(2-px_2)^{-½}$, and thus $0 = (2-px_2)^{½} - p(x_2)^{½}$, or $0 = (2-px_2) - p^2(x_2)$ and $x_2 = \frac{2}{p(1+p)}$ and $y_2 = 2-px_2 = \frac{2p}{1+p}$.

b. The marginal rate of substitution is $-\left.\frac{dy}{dx}\right|_{u\ cons\tan t} = \sqrt{\frac{y}{x}}$. Thus, the contract curve sets $\sqrt{\frac{y}{x}} = \sqrt{\frac{2-y}{1-x}}$, or y=2x, which is the diagonal of the Edgeworth box. A point on the contract curve gives (x,2x) to the first consumer and (1-x, 2-2x) to the second. This gives the first consumer utility of $(1+\sqrt{2})\sqrt{x}$ and the second consumer $(1+\sqrt{2})\sqrt{1-x}$ The first consumer can insist on at least 1, the second on at least $\sqrt{2}$, which yields constraints of $(1+\sqrt{2})\sqrt{x} \geq 1$ and $(1+\sqrt{2})\sqrt{1-x} \geq \sqrt{2}$, or

$$4\sqrt{2}-5 = 1-\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)^2 \geq x \geq \frac{1}{(1+\sqrt{2})^2} = 3-2\sqrt{2}$$

c. Using the demands from part a, $1 = x_1 + x_2 = \frac{1}{(1+p)} + \frac{2}{p(1+p)} = \frac{p+2}{p(1+p)}$, or $p(1+p) = 2+p$, or $p = \sqrt{2}$.

4. (20 points) A risk averse person has utility of wealth x of $u(x) = x^a$ and a current wealth level of \$100,000. What is the most the person would pay for insurance against a 50% chance of a \$10,000 loss. (Don't include the expected loss of \$5000, but the premium only.)

The willingness to pay for insurance is the amount x in the certainty equivalent equation

$$(½(100000+90000)-x)^a = ½(100000^a + 90000^a).$$

Thus, $x = 95000 - (½(100000^a + 90000^a))^{1/a}$

This yields \$131.67 at a=0, and is less for any a in (0,1).

5. (10 points) Find all the equilibria of the following game:

		Woman	
		Baseball	Ballet
Man	Baseball	(2,3)	(1,0)
	Ballet	(0,1)	(3,2)

There are two pure strategies: (Baseball, Baseball) and (Ballet, Ballet). In addition, there is a mixed strategy with the man going to baseball ¼ of the time, and the woman going to baseball ¾ of the time.

The following questions can be adequately answered with short answers, no more than two or three lines.

6. (5 points, short answer) Incoming college students assigned to dorm rooms with roommates that drank alcohol the previous year obtain 10% lower GPAs on average, when compared to those with roommates that did not drink alcohol. How would use economic analysis to improve the assignment of students to dorm rooms?

This is an externality – the drinking roommate lowers someone else's GPA. We could tax drinking, we could create a market for roommates with payments by, or taxes on, drinking roommates. We could ban drinking (didn't work last time).

7. (5 points, short answer) In an antitrust analysis, how would you determine if sprite is in a market containing colas?

Start with colas and consider whether a monopolist would find it profitable to increase price a significant amount, or would consumers substitute to sprite to such an extent that the increased prices were not profitable.

8. (5 points, short answer) In a setting where incentives matter, does an increase in risk reduce the efficiency of production?

Yes. An increase in risk should, other things equal, reduce the strength of incentives, reducing the output.