Ec 11 Practice Midterm Examination
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CALTECH

1. James and Joan work in a medical insurance firm. In T hours, James can process $T^{0.8}$ normal claims, and ¼ T new applications. In T hours, Joan can process $T^{0.5}$ normal claims, and ½ T new applications. There are 40 new applications and 10 normal claims to process. If they work the same number of hours, what is the minimum time they need to work? Hint: This problem is not designed so that the numbers work out evenly.

2. A shoe factory has a fixed cost of $5 million, and a marginal cost the q^{th} unit of Max[$10, q/100,000]. What are average total costs? What is the shutdown point of this factory? What is its short run supply? What is the long run competitive supply of shoes?

3. You have one child born today. You want to send your child to private college. Figuring that a college will cost about $50,000 per year in today's dollars for four years, how much do you need to save per year, if you expect 5% on your investments to accomplish this task? (Assume that payments are made on the child's birthday and that they go to college on their 18^{th} birthday. Moreover, you can save for the college during the four years they are in college.)

4. If the Ricardian theory is true, what kinds of goods should California export outside the state, and what should it import? (What factors are in short supply, and what are in abundance in the state?) How well does the theory hold up?

5. For a consumer with Cobb-Douglas utility, what is the formula for the elasticity of demand for a good?

Answers

1. Let N be the number of claims processed by James, so that Joan processes 10 – N. Let H be the number of hours they work. The N claims processed by James take $N^{1/.8}$ hours, so he has $H - N^{1/.8}$ hours to process new applications, resulting in ¼$\left(H - N^{1/.8}\right)$ new applications. Joan takes $(10 - N)^{1/.5}$ to process her claims, so she processes ½$\left(H - (10 - N)^{1/.5}\right)$ new applications. H must be enough to process the 40 new applications, so

$$40 = \tfrac{1}{4}\left(H - N^{1/.8}\right) + \tfrac{1}{2}\left(H - (10 - N)^{1/.5}\right)$$

$$160 = H - N^{1/.8} + 2H - 2(10 - N)^{1/.5}$$

$$\mathrm{H} = \frac{1}{3}\left(160 + N^{1/.8} + 2(10 - N)^{1/.5}\right)$$

The minimum time occurs when this is minimized over N, which occurs at N = 9.45 and H = 59.06.

2. A shoe factory has a fixed cost of $5 million, and a marginal cost the q[th] unit of Max[$10, q/10,000]. What are average total costs? What is the shutdown point of this factory? What is its short run supply? What is the long run competitive supply of shoes?

Note that marginal costs are $10 up to 100,000 units, then q/10000 for q > 100,000. Thus total costs c(q) are

$$c(q) = \begin{cases} 5{,}000{,}000 + 10q & \text{if } q \le 100{,}000 \\ 6{,}000{,}000 + \dfrac{(q - 100000)^2}{20000} & \text{if } q > 100{,}000 \end{cases}$$

Average costs are c(q)/q. The shutdown point occurs at $10, which is the minimum of variable cost.

Short run supply is marginal cost, which is

$$c'(q) = \begin{cases} 10 & \text{if } q \le 100{,}000 \\ \dfrac{q - 100000}{10000} & \text{if } q > 100{,}000 \end{cases}$$

Note that average costs are decreasing for q<100,000, so that minimum average cost occurs for $q \ge 100{,}000$. Minimum average cost occurs at $q = 100000\sqrt{13} = 360{,}555$ with an average cost of $10\left(\sqrt{13} - 1\right) \approx \$26.06$. This is the long run supply price of a competitive industry.

3. The present value of the college costs is

$$\$50000 \times (1.05)^{-18} \times \left(1 + 1.05^{-1} + 1.05^{-2} + 1.05^{-3}\right) = \$77{,}354.30$$

The annual amount to set aside, then, satisfies

$$\$77{,}354.30 = A \times \left(1 + \frac{1}{.05}\left(1 - \frac{1}{(1.05)^{21}}\right)\right)$$

from the formula for mortgages (note here the difference that a payment is made immediately, accounting for the extra 1. Thus the annual payment is

$$A = \frac{\$77{,}354.30}{1 + \frac{1}{.05}\left(1 - \frac{1}{(1.05)^{21}}\right)} = \$5{,}596.81$$

4. California has a relatively highly-educated workforce and abundant and fertile land with a long growing season. Thus exports of skill-intensive goods, like movies and semiconductor designs, along with agricultural goods, are consistent with Ricardian theory. California is short on water, so production of water-intensive goods like rice and cotton are inconsistent with the theory.

5. Demand for Cobb-Douglas is in the form $x = \dfrac{\alpha M}{p_X}$, so $\varepsilon = -\dfrac{px'}{x} = 1$.